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Universal aspects of localized excitations in graphene

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Abstract—Unique features of nonlinear solitary plasmon excitations in two and three dimensional massless Dirac fluids, with respect to their normal Fermi counterparts, are explored using the Bernoulli pseudopotential method. It is revealed that graphene, as a two dimensional Dirac fluid, possesses some unique characteristics with respect to the propagation of the localized plasmon excitations, which is absent in other ordinary solids. It is also shown that the Mach number limit below/above, which the localized solitary/periodic excitations propagate in a monolayer graphene has a universal value independent of the other environmental parameters such as the electron number-density and the ambient temperature. The amplitude of nonlinear solitary or periodic waves is also remarked to be independent of such parameters and depend only on the Mach-number value of the solitary or periodic excitations. These unique hydrodynamic wave features of the massless Dirac fluid are attributed to the remarkable photon-like linear energy dispersion in Dirac points of graphene material. © 2013 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4818707]

I. INTRODUCTION

Graphene, since its recent experimental discovery in 2004, has become one of the most exciting and technologically appealing fields of research. During the last few years, a large number of research material have appeared on investigation of exceptional physical properties of graphene. The perfect atom-thick monolayer arrangements of carbon atoms in a packed honeycomb lattice and interaction of the electrons with this unique atomic structure makes the charge carriers in graphene move with an effective zero-mass in order to mimic photon-like dispersion in the low energy excitation of the lattice structure. Study of the electromagnetic response of the graphene reveals that the electromagnetic interactions with graphene structure are strongly nonlinear for a broad range of the electromagnetic spectrum. On the other hand, the investigation of nonlinear dynamic conductivity of graphene reveals that the nonlinear optical characteristics of graphene may have promising applications in future advanced terahertz sources and sensors technologies (terahertz gap technology). The recently predicted frequency multiplication property of graphene has also desirable applications in fabrication of the low-power terahertz electromagnetic radiation and detection devices. Moreover, graphene shows distinguished differences with semiconductors regarding the plasmon excitation resonance due to the unique zero energy-gap character. Recent studies show that propagation of the strongly localized electromagnetic surface transverse electric and magnetic plasmon and solitary excitations are possible in graphene.

The extraordinary properties of graphene are indebted to the fact that the chemical potential crosses exactly the Dirac point so that the low energy electrons behave like photons, except that they move with constant subluminal Fermi velocities, \( v_F \approx c/300 \) which is independent of the charier number-density, unlike ordinary Fermi solids. It has been shown that the thermodynamic property of a Dirac plasma is strongly affected by the electron energy dispersion relation. Such extraordinary behavior makes graphene an ideal material for the relativistic quantum electrodynamic tabletop experiments. Graphene could also be used as a stand-alone particle-physics laboratory to do research in the quantum field theories that describe the remarkable high energetic electron-matter interactions, without the need for expensive particle accelerators. More importantly, graphene makes it possible to understand the behavior of matter under extreme conditions, like high pressure and temperature, which are not attainable on the earthly environments. It is well known that the electrons of matter become relativistic under the high external pressure. This is so called the relativistic degeneracy and is known to be a novel feature of stellar cores leading to some extraordinary effects of compact stars like white dwarfs and neutron stars. In the extreme relativistic degeneracy regime, the electrons disperse similar to photons, just as they do in graphene. This is shown to lead to the fundamental change in the equation of state of the relativistically degenerate matter and phenomena like stellar core collapse and supernova. Recent experiments have revealed that the relativistic graphene electrons can lead to the strange effect of the atomic collapse. This behavior might as well be related to the stellar core collapse by the gravitationally induced extreme-relativistic electrons in the critical-density (critical-mass) white dwarfs.

One of the important tools for investigation of the physical properties of ionized matter is via its linear and nonlinear wave propagation due to the density or electrostatic potential perturbations. Hydrodynamic analysis is one of the most powerful method to investigate such characteristics of matter as the linear response, diverse instabilities, shock formation, matter-wave interaction, charge shielding, ion structure formation, polarizability, conductivity and optical properties, etc.
early theoretical development of hydrodynamic theory of dense plasmas, is the extension of quantum hydrodynamic (QHD) and quantum magnetohydrodynamic (QMHD) theories to include the electron spin, magnetization, and relativistic effects, during the recent few years. Among the approximate tools to investigate the small amplitude nonlinear perturbations is the reductive perturbation technique known as the multiscales, which relies on the limiting approximation of the perturbation amplitude being much smaller than that of the perturbed quantity itself, \( \delta Q \ll Q \). On the other hand, the pseudopotential methods, such as the Sagdeev and the Bernoulli, are perfect tools for investigation of the arbitrary amplitude nonlinear solitary, shock and periodic excitations in a plasma. In current research by using appropriate quantum models, we will employ the pseudopotential method to investigate the nonlinear plasmon dynamics of Dirac and Fermi liquid and reveal some unique aspects of graphene as an example of the two dimensional massless Dirac fluid, compared to that of the Fermi counterparts. The paper is organized as follows. The QHD models are introduced for the Fermi and Dirac plasma cases in Sec. II. The reduced hydrodynamic equations in the stationary frame are introduced in Sec. III, and they are solved together to find the solitary and periodic nonlinear wave solutions using the Bernoulli pseudopotential method. We analyze the findings and make some discussion in Sec. IV and give a summary in Sec. V.

II. QUANTUM HYDRODYNAMICS MODEL AND THE GOVERNING EQUATIONS

To investigate the nonlinear wave dynamics of a quasi-neutral plasma with fixed ion background and quantum dense electron fluid with relativistic effects, the following closed QHD model will be used:

\[
\begin{align*}
\frac{\partial n}{\partial t} + \nabla \cdot (nm) &= 0, \\
\frac{\partial u}{\partial t} + (u \cdot \nabla)u &= \frac{en}{(p + \varepsilon)^{1/2}} \left[ \nabla \phi + \beta \times B + \beta (\beta \cdot \nabla \phi) \right] \\
&\quad - \frac{c^2}{p + \varepsilon} \left[ \left( \nabla P + \frac{\partial \varepsilon}{c \partial t} \right) \nabla \phi \right], \\
\Delta \phi(r) &= 4\pi e(n - n_0),
\end{align*}
\]

in which \( \beta = u/c, \gamma = 1/\sqrt{1 - \beta^2} \). Parameters \( n_0, P, \) and \( \varepsilon \) denote the equilibrium electron (ion) number-density, the hydrodynamic pressure and the plasma energy density, respectively, with the other symbols having their usual physical meanings. The above QHD model can be applied for both cases of non-relativistic quantum Fermi liquid and the relativistic massless Dirac fluid, such as graphene. The total pressure \( P \) of the Eq. (1) can be decomposed into the classical and quantum parts in the weak relativistic limit \( P \ll \varepsilon = mn \), where, \( P = P_c + P_q \), gives rise to the classical part representing the ordinary Fermi degeneracy pressure and the quantum part includes the Bohm quantum diffraction potential \( V_B = (\hbar^2/2m_e)\Delta \sqrt{n}/\sqrt{n} \). In the case of unmagnetized plasma and with the assumption, \( n \leq n_F \ll c/(\gamma \simeq 1) \), one obtains the following relations:

\[
\begin{align*}
\frac{\partial n}{\partial t} + \nabla \cdot (nm) &= 0, \\
\frac{\partial u}{\partial t} + (u \cdot \nabla)u &= \frac{e\gamma}{m_e} \nabla \phi - \frac{1}{m_e n} \nabla P + \frac{\hbar^2}{2m_e} \nabla \left( \frac{\Delta \sqrt{n}}{\sqrt{n}} \right), \\
\Delta \phi(r) &= 4\pi e(n - n_0),
\end{align*}
\]

where \( P_F \) represents the Fermi degeneracy pressure. In what follows, we will ignore the effect of quantum diffraction caused by the Bohm quantum force (we take \( V_B \simeq 0 \), for simplicity, as it is minor compared to the quantum degeneracy force for the solid density plasmas. On the other hand, for the case of the Dirac fluid, the Fermi speed \( v_F \) is \( \simeq c/300 \) and the condition \( \beta \ll 1 \) applies. However, due to the different energy relation \( E = \hbar^2 k^2/2m_e \), the equation of states for Fermi and Dirac cases, hence, the quantum pressure in Eq. (1) would completely differ and the weak relativistic assumption does not apply to Dirac case, anymore. Therefore, we find the following simplified model for the case of the massless Dirac fluid:

\[
\begin{align*}
\frac{\partial n}{\partial t} + \nabla \cdot (nm) &= 0, \\
(P_D + \varepsilon) \left[ \frac{\partial u}{\partial t} + (u \cdot \nabla)u \right] &= en\gamma^2 \nabla \phi - \varepsilon \nabla P_D, \\
\Delta \phi(r) &= 4\pi e(n - n_0),
\end{align*}
\]

where \( P_D \) is the quantum pressure for Dirac electron fluid to be calculated as follows. Note that in both models, we have assumed that the plasma is completely degenerate (the zero-temperature assumption). Such assumption is valid if \( \lambda_D < a \), where \( \lambda_D = h/\sqrt{2\pi m_e k_B T_p} \) is the DeBroglie thermal wavelength, where \( h, k_B, \) and \( T_p \) are the Plank constant, the Boltzmann constant, and the particle temperature, respectively, and \( a \) is the inter-fermion or inter-ion spacings in the quasineutral plasma, which is also related to the equilibrium plasma number density, \( n_0 \). Simple calculations reveal that for two and three dimensional electron gasses the complete degeneracy condition establishes with \( n_0 > 10^{11} \text{cm}^{-2} \) and \( n_0 > 10^{18} \text{cm}^{-3} \) for \( T_p = 300 \text{K} \), respectively, which is readily satisfied for the monolayer graphene with the typical electron density of \( n_0 \approx 10^{13} \text{cm}^{-2} \). However, the assumption of a well-defined Fermi wavevector, \( k_F \) can be valid with the standard definition, \( n_0 = \int_0^{k_F} d^3k/(2\pi)^3 \), where, \( D \) is the system dimensionality. In order to use the Fermi and Dirac QHD models given by Eqs. (2) and (3), it is required to calculate the corresponding pressures, \( P_F \) and \( P_D \). For completeness and for comparison, we calculate the pressures for both two and three dimensional cases. Assuming, the complete degeneracy, the energy density is obtained via \( \varepsilon = \int_0^{k_F} E(k)d^3k \), where, \( E(k) \) is the corresponding energy dispersion relation, for the general dimensionality parameter, \( D \). Now applying the thermodynamical identity, \( P = n[(\partial \varepsilon/\partial n) - \varepsilon] \), following pressure components are easily obtained for the two and three dimensional Fermi and Dirac fluids.
\[ P_{2D} = \frac{\sqrt{2\pi}}{3} v_F h n^{3/2}, \quad P_{3D} = \frac{(3\pi^2)^{4/3}}{12\pi^2} v_F h n^{4/3}, \]
\[ P_{2F} = \frac{\pi h^2 n^2}{2m_e}, \quad P_{3F} = \frac{(3\pi^2)^{2/3} h^3 n^{5/3}}{5m_e}. \quad (4) \]

Note that, for the Dirac electron fluid, the relation \( P = \rho / D \) is valid in general, which is consistent with the photon-like pressure expected for a relativistic Dirac electron gas. It is also to be noted that, the QHD model for Dirac fluid is independent of the electron mass, as it should be. Now, we are in a position to solve the model equations for the desired nonlinear stationary wave solution.

III. NONLINEAR STATIONARY WAVE SOLUTION FOR FERMI AND DIRAC ELECTRON GASES

In order to solve the model equations in the stationary frame, we introduce the change of variable within a one-dimensional scheme, \( \xi = x - Mt \) with \( M \) being the normalized phase-speed of the nonlinear waves, so called the Mach-number (the normalization values are the speed of light in vacuum and the Fermi speed for the cases of Dirac and Fermi quantum fluids, respectively). The reduced model equations, in which particle velocity and density are scaled as \( u \rightarrow cu \) for Dirac case, \( u \rightarrow v_F u \) for Fermi case and \( n \rightarrow n_0 n \), can be written, for instance, for the Dirac quantum fluids, as

\[ [P_D(n) + \varepsilon] \frac{-M \partial_t n + \partial_t (nu)}{en_0 n} = 0, \quad [P_D(n) + \varepsilon] \frac{-M \partial_t u(n) + \partial_t (u(n)^2 / 2)}{en_0 n} = \partial_{\xi} \phi, \quad \partial_{\xi} \phi = 4\pi e n_0 (n - 1). \quad (5) \]

Also, for the Fermi electron fluids, the set of the reduced equation becomes

\[ \frac{mv_F^2}{e} \frac{-M \partial_t n + \partial_t (nu)}{en_0 n} = 0, \quad \frac{mv_F^2}{e} \frac{-M \partial_t u(n) + \partial_t (u(n)^2 / 2)}{en_0 n} = \partial_{\xi} \phi, \quad \partial_{\xi} \phi = 4\pi e \mu_0 (n - 1), \quad (6) \]

where \( P_D \) and \( P_F \) are given by Eq. (4). Now, the momentum and continuity equations of Eqs. (5) and (6) can be solved together in each model, assuming the standard boundary conditions at equilibrium, \( \lim_{\xi \rightarrow \pm \infty} \{n, u, \phi\} = \{n_0, 0, 0\} \), in order to obtain the electrostatic potentials in terms of the number densities, as

\[
\begin{align*}
\phi(n) &= 1 + \frac{2}{3} M^2 - \frac{M^2 (2 - 6n)}{3n^{2/3}} - \frac{2M^2}{\sqrt{n}} + \sqrt{n}, \quad A_{2D} = \frac{\hbar c \sqrt{2\pi n_0}}{(300e)} \quad 2D \text{ Dirac} \\
\phi(n) &= 1 + \frac{9M^2}{20} \frac{9M^2}{8n^{2/3}} - \frac{9M^2 (5n - 2)}{40n^{5/3}} - \frac{M^2 (2 - 6n)}{3n^{2/3}}, \quad A_{3D} = 3^{1/3} \hbar n_0^{1/3} n^{2/3} / \mu_{0} (300e) \quad 3D \text{ Dirac} \\
\phi(n) &= \frac{M^2}{n} - \frac{M^2}{n^{3/2}} + B_{2F} (1 - n), \quad A_{2F} = m_e v_F^2 / e, \quad B_{2F} = \pi \hbar^2 n_0 / (m_e v_F^2) \quad 2D \text{ Fermi}
\end{align*}
\]

In the standard Sagdeev approach, one has to solve Eqs. (7) for densities and insert into the Poisson’s equation. However, as it is apparent, in this case, it is not possible to solve the equations analytically for \( n(\phi) \). Therefore, we instead employ the Bernoulli method to find the appropriate pseudopotential describing the propagation of the nonlinear excitations in both cases. The Bernoulli pseudopotential approach has been developed and thoroughly discussed by Dubinov et al. To this end, we define an auxiliary function \( F(n) = d\phi(n) / dn \), so that

\[ \frac{d^2 \phi}{dz^2} = F(n) \frac{d^2 n}{dz^2} + \frac{dF(n)}{dn} (\frac{dn}{dz})^2 = 4\pi e n_0 (n - 1). \quad (8) \]

Hence, the desired Bernoulli pseudopotentials describing the nonlinear and periodic excitations are given by, \( U_B(n) = 4\pi e n_0 \int F(n) (n - 1) dn \). The \( F(n) \) functions for Eqs. (7) are given as follows:

\[
\begin{align*}
F_{2D}(n) &= A_{2D} [2M^2 (4 + 9n^{1/6} + 6n) - 9n^{7/6}] / (18n^{5/3}) \quad 2D \text{ Dirac} \\
F_{3D}(n) &= A_{3D} (9M^2 - 4n^2) / (12n^{8/3}) \quad 3D \text{ Dirac} \\
F_{2F}(n) &= A_{2F} (M^2 / n^3 - B_{2F}) \quad 2D \text{ Fermi}
\end{align*}
\]

and the corresponding Bernoulli pseudopotentials, \( U_B(n) \) are

\[ \begin{align*}
U_{2D}(n) &= 2A_{2D} m_0 n_0 \pi e (M^2 (4 + 12n^{1/6} - 27n^{2/3} - 4n + 12n^{7/6} + 3n^2) - 2[2 + (n - 3) \sqrt{n}] n^{2/3}) / (3n^{2/3}) \\
U_{3D}(n) &= A_{3D} m_0 n_0 \pi e [M^2 (2 - 5n + 3n^{5/3}) - 10(3n^{5/3} - 4n^2 + n^3)] / (10n^{5/3}) \\
U_{2F}(n) &= 2A_{2F} m_0 n_0 \pi e (n - 1)^2 (M^2 - B_{2F} n^2) / n^2
\end{align*} \]
It is clearly evident that \( U_B(0) = dU_B(n)/dn(n = 0) = 0 \) as is expected. For the second derivative of the pseudopotential being less/greater than zero it is possible to have solitary/periodic waves propagate in the plasma. Also, for simple roots of the pseudopotential being less/greater than unity one obtains rarefactive/compressive solitary or periodic waves in the stationary limit. On the other hand, direct evaluation of the second derivatives of the pseudopotentials for different cases leads to the following critical Mach-number value below/above, which solitary/periodic waves can exist

\[
\begin{align*}
\text{for 2D Dirac:} & \quad d_{\text{nn}}U_{2D}(n)|_{n=0} = 2A_{2D}n_0\pi e(38M^2 - 9)/9 \\
\text{for 3D Dirac:} & \quad d_{\text{nn}}U_{3D}(n)|_{n=0} = A_{3D}n_0\pi e(9M^2 - 4)/3 \\
\text{for 2D Fermi:} & \quad d_{\text{nn}}U_{2F}(n)|_{n=0} = 4A_{2F}n_0\pi e(M^2 - B_{2F})
\end{align*}
\]

\[ (11) \]

It is readily found that for 2D (graphene) and 3D Dirac plasmas, the universal values of the critical Mach-numbers are

- For 2D Dirac, \( M^2_{2D} = 3/\sqrt{38} \)
- For 3D Dirac, \( M^2_{3D} = 2/3 \)

and for the case of 2D Fermi, we have \( M^2_{2F} = \sqrt{B_{2F}} = h\sqrt{\pi n_0}/(m_e v_F) \). Also, by further mathematical evaluation it is observed that \( \lim_{n\to 0} U_B(n) = +\infty \) and \( \lim_{n\to \infty} U_B(n) = -\infty \) is always valid for all the cases under investigation. This indicates that the solitary waves are of rarefactive type for all the cases. In Sec. IV, we are going to discuss the interesting properties of the nonlinear excitations in Dirac plasmas (of which the 2D graphene is a unique example) numerically and compare those with that of the well-studied Fermi case.

**IV. NUMERICAL ANALYSIS AND DISCUSSION**

As it was previously noticed, only rarefactive solitons are allowed to propagate in both Dirac and Fermi quantum electron fluids. The maximum Mach number corresponding to the solitary waves in these plasmas are also given by Eqs. (11). It is easily remarked that, the critical Mach-number values (maximum normalized phase-speed corresponding to the solitary waves) for Dirac plasmas are universally constant \( \{M_c = \{0.49, 0.67\} \) for 2D and 3D cases, respectively), meaning that these values are independent of other plasma parameters. The impurity contamination, doping or change in the temperature is known to greatly influence the plasma number density. However, the critical Mach-number for the case of Dirac plasma, particularly a monolayer graphene, is not influenced by the change in these parameters. Also, comparing the pseudopotential solutions given in Eqs. (10), it is revealed that the corresponding roots for the cases of 2D and 3D Dirac fluids are independent of the plasma number density, while for the 2D Fermi case, this is not the case. However, roots of the pseudopotentials depend on the value of the wave Mach-number, for all the cases.

The variation of the normalized Bernoulli pseudopotential profiles is shown for different plasma parameters for the cases of a monolayer graphene in Fig. 1. This figure depicts

![Graph](image-url)
the Bernoulli pseudopotential profiles describing both solitary (Figs. 1(a) and 1(b)) and periodic (Figs. 1(c) and 1(d)) solutions. It is clearly observed from Figs. 1(a) and 1(c) that for graphene, the solitary and periodic pseudopotential roots are independent of the plasma number density value, $n_0$. On the other hand, it is remarked from Figs. 1(b) and 1(d) that the value of solitary and periodic pseudopotential roots increase with increase in the Mach number value. In Fig. 2, we have shown the solitary profiles for the case of graphene and with the same parameters used in Fig. 1. From Fig. 2(a), it is revealed that for monolayer graphene the solitary plasmon excitation is of rarefactive type whose amplitude is independent of the electron number density (hence, independent of other parameters like doping, impurity concentration, and the plasma temperature). This is a unique feature caused by the remarkably distinguished energy dispersion relation of low-energy exciting electrons in graphene at the Dirac points. Such unique photon-like characteristics has been shown to produce many other astonishing physical properties, which remarkably distinguished energy dispersion relation of low-plasma temperature. This is a unique feature caused by the dimensionality. It was found that only rarefactive solitons are allowed to propagate in Fermi and Dirac electron fluids and that Dirac fluid of which graphene is a distinct example possesses unique feature. It was further revealed that the critical Mach-number below/above, which solitary/periodic nonlinear waves are allowed to propagate in Dirac plasmas is independent from electron number-density and many other plasma parameters, unlike that of the Fermi counterparts. This feature also applies to the amplitude of a solitary wave propagating in a Dirac quantum fluid. However, it was remarked that in graphene the width of the solitary waves are significantly affected by the change in the plasma number density.

**V. CONCLUSION**

We investigated the possibility of the propagation of nonlinear localized solitary and periodic plasmon excitations in Fermi and Dirac quantum electron fluids with different dimensionality. It was found that only rarefactive solitons are allowed to propagate in Fermi and Dirac electron fluids and that Dirac fluid of which graphene is a distinct example possesses unique feature. It was further revealed that the critical Mach-number below/above, which solitary/periodic nonlinear waves are allowed to propagate in Dirac plasmas is independent from electron number-density and many other plasma parameters, unlike that of the Fermi counterparts. This feature also applies to the amplitude of a solitary wave propagating in a Dirac quantum fluid. However, it was remarked that in graphene the width of the solitary waves are significantly affected by the change in the plasma number density.

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